

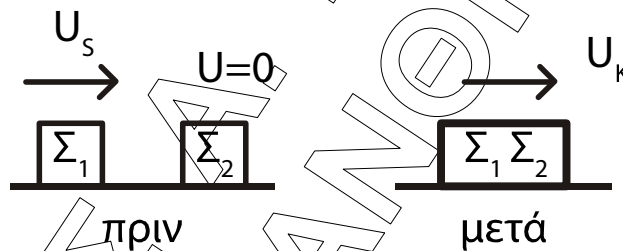
**ΦΥΣΙΚΗ**  
**ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΥ**  
**12 ΙΟΥΝΙΟΥ 2019**  
**ΑΠΑΝΤΗΣΕΙΣ**

**ΘΕΜΑ Α**

- A1. β  
 A2. γ  
 A3. α  
 A4. γ  
 A5. α) → Λ, β) → Σ, γ) → Λ, δ) → Σ, ε) → Σ

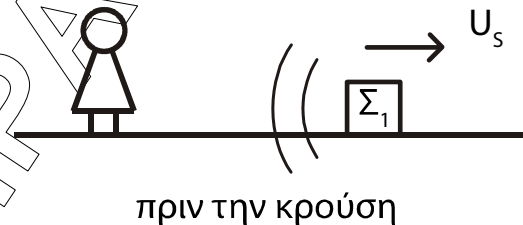
**ΘΕΜΑ Β**

B1. Πλαστική κρούση  $\Sigma_1 - \Sigma_2$ .

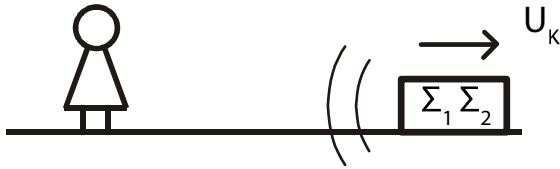


Α.Δ.Ο.  $\vec{P}_{\text{πριν}} \equiv \vec{P}_{\text{μετά}}$

$$\Rightarrow m v_s + 0 = (m + m) v_k \Rightarrow \left. \begin{array}{l} v_k = \frac{v_s}{2} \\ v_s = \frac{v_{\eta\zeta}}{20} \end{array} \right\} \frac{v_{\eta\zeta}}{40} \quad (1)$$



$$f_1 = \frac{v_{\eta\zeta} + 0}{v_{\eta\zeta} + v_s} f_s = \frac{v_{\eta\zeta}}{v_{\eta\zeta} + v_s} f_s \quad (2)$$



μετά την κρούση

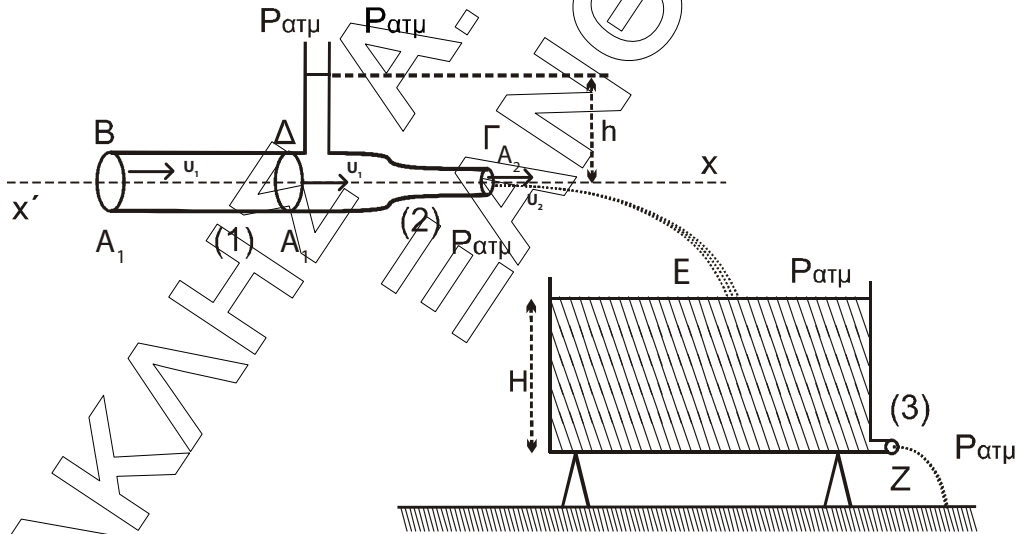
$$f_2 = \frac{v_{\eta\zeta}}{v_{\eta\zeta} + v_k} f_s = \frac{v_{\eta\zeta}}{v_{\eta\zeta} + \frac{v_s}{2}} f_s \quad (3)$$

Από  $\frac{(2)}{(3)} \Rightarrow \frac{f_1}{f_2} = \frac{\frac{v_{\eta\zeta}}{v_{\eta\zeta} + v_s} f_s}{\frac{v_{\eta\zeta}}{v_{\eta\zeta} + v_k} f_s} = \frac{v_{\eta\zeta} + v_k}{v_{\eta\zeta} + v_s} =$

$$= \frac{v_{\eta\zeta} + \frac{v_{\eta\zeta}}{40}}{v_{\eta\zeta} + \frac{v_{\eta\zeta}}{20}} = \frac{41}{42}$$

Άρα σωστό το (ii)

B2.



Εξ. συνέχ. Από Δ → Γ

$$\Pi_1 = \Pi_2 \Rightarrow A_1 v_1 = A_2 v_2 \Rightarrow 2A_2 \cdot v_1 = A_2 v_2 \Rightarrow \underline{v_2 = 2v_1} \quad (1)$$

Bernoulli: Δ → Γ

$$\left. \begin{aligned} P_{\Delta} + \frac{1}{2}\rho v_1^2 &= P_2 + \frac{1}{2}\rho v_2^2 \\ \text{Στον κατακόρυφο σωλήνα} &\Rightarrow P_{\text{atm}} + \rho gh + \frac{1}{2}\rho v_1^2 = P_{\text{atm}} + \frac{1}{2}\rho v_2^2 \Rightarrow \\ P_{\Delta} &= P_{\text{atm}} + \rho gh \end{aligned} \right\}$$

$$\Rightarrow gh + \frac{1}{2}v_1^2 = \frac{1}{2}v_2^2 \Rightarrow gh + \frac{1}{2}\frac{v_2^2}{4} = \frac{1}{2}v_2^2 \Rightarrow$$

$$\Rightarrow \frac{3}{8}v_2^2 = gh \Rightarrow v = \sqrt{\frac{8}{3}gh} \quad (2)$$

$$\left. \begin{aligned} \text{Στο δοχείο η επιφάνεια σταθερή σε ύψος (H)} \\ \Pi_2 = \Pi_3 \end{aligned} \right\} \Rightarrow A_2 v_2 = A_3 v_3 \Rightarrow$$

άρα:

$$\Rightarrow A_2 v_2 = \frac{A_2}{2} v_3 \Rightarrow v_3 = 2v_2$$

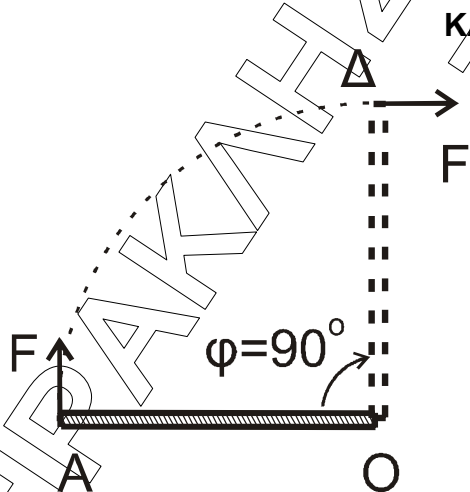
Bernoulli: E  $\rightarrow$  Z

$$P_{\text{atm}} + \rho gH + 0 = P_{\text{atm}} + \frac{1}{2}\rho v_3^2 + 0 \Rightarrow gH = \frac{1}{2}4v_2^2 \Rightarrow$$

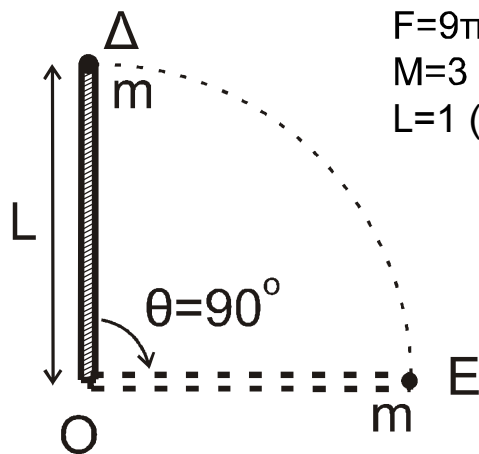
$$\Rightarrow gH = 2v_2^2 \Rightarrow v_2 = \sqrt{\frac{gH}{2}} \text{ από την (2)} \Rightarrow \sqrt{\frac{8}{3}gh} = \sqrt{\frac{gH}{2}} \Rightarrow \frac{h}{H} = \frac{3}{16}$$

Σωστό (iii)

B3



Σχήμα 4



Σχήμα 5

$$\begin{aligned} F &= 9\pi \text{ (N)} \\ M &= 3 \text{ (Kg)} \\ L &= 1 \text{ (m)} \end{aligned}$$

Για την κίνηση A  $\rightarrow$  Δ από το Θ.Μ.Κ.Ε. ισχύει:

$$\Delta K = \Sigma W \Rightarrow \frac{1}{2} \cdot I_0 \cdot \omega_\Delta^2 = (F \cdot L) \frac{\pi}{2} \Rightarrow \frac{1}{2} \cdot \frac{1}{3} \cdot M \cdot L^2 \cdot \omega_\Delta^2 = F \cdot L \cdot \frac{\pi}{2} \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cdot \frac{1}{3} \cdot 3 \cdot 1 \cdot \omega_\Delta^2 = 9\pi \cdot 1 \cdot \frac{\pi}{2} \Rightarrow \omega_\Delta = 3\pi \text{ rad/s}$$

Από Α.Δ.Σ. στην κρούση στο (Δ) ισχύει:

$$\vec{L}_{\text{πριν}} = \vec{L}_{\text{μετά}} \Rightarrow I_0 \cdot \omega_\Delta = I'_0 \cdot \omega'_\Delta \Rightarrow \omega'_\Delta = \frac{I_0 \cdot \omega_\Delta}{I'_0} \quad (1)$$

$$\text{Όμως } I'_0 = \frac{1}{3} M \cdot L^2 + mL^2 = \frac{3 \cdot 1^2}{3} + 1 \cdot 1^2 = 2 \text{ Kgm}^2 \quad (2)$$

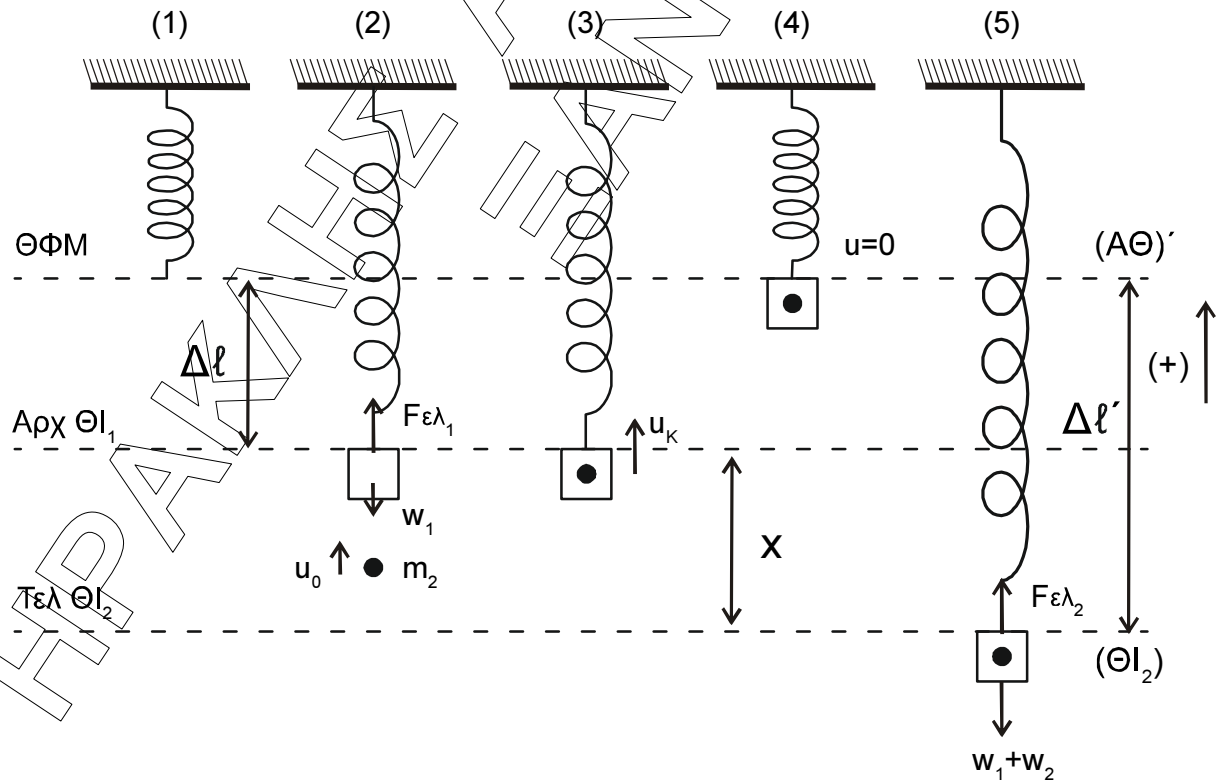
$$\text{Από (1) και (2) έχουμε: } \omega'_\Delta = \frac{\frac{ML^2}{3} \cdot \omega_\Delta}{2} = \frac{\frac{3 \cdot 1}{3} \cdot 3\pi}{2} = \frac{3\pi}{2} \text{ rad/s}$$

Για τον χρόνο  $t_{\Delta \rightarrow E} = t$  έχουμε

$$\Delta\Theta = \omega'_\Delta \cdot t \Rightarrow \frac{\pi}{2} = \frac{3\pi}{2} \cdot t \Rightarrow t = \frac{1}{3} \text{ (s)}$$

Άρα σωστό είναι το (ii).

### ΘΕΜΑ Γ



**Γ1.** Για την αρχική Θ.Ι1 σχήμα (2) ισχύει:

$$\Sigma F_y = 0 \Rightarrow W_1 = F_{ελ1} \Rightarrow m_1 \cdot g = K \cdot \Delta l \Rightarrow K = \frac{10}{0,05} = 200 \text{ N/m}$$

Για την τελική Θ.Ι2, σχήμα (5)

$$\Sigma F_y = 0 \Rightarrow W_1 + W_2 = F_{ελ2} \Rightarrow (m_1 + m_2) \cdot g = K \cdot \Delta l' \Rightarrow 20 = 200 \cdot \Delta l' \Rightarrow \Delta l' = 0,1 \text{ m}$$

Άρα το πλάτος ΑΘ'-ΘΙ2:  $\Delta l' = A = 0,1 \text{ m}$

**Γ2.**  $x = \Delta l' - \Delta l = 0,05 \text{ m}$

**ΑΔΕΤ:**

$$K + U = E_T \Rightarrow \frac{1}{2} \cdot (m_1 + m_2) \cdot v_K^2 + \frac{1}{2} \cdot k \cdot x^2 = \frac{1}{2} \cdot k \cdot A^2 \Rightarrow 2 \cdot v_K^2 + 200 \cdot 0,05^2 = 200 \cdot 0,1^2 \Rightarrow$$

$$\Rightarrow v_K^2 = 1 - 0,25 \Rightarrow v_K = \sqrt{0,75} = \frac{\sqrt{3}}{2} \text{ m/s}$$

$$\text{Από: } \overrightarrow{P}_{\text{πριν}} = \overrightarrow{P}_{\text{μετά}} \Rightarrow m \cdot v_o = 2 \cdot m \cdot v_K \Rightarrow v_o = \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3} \text{ m/s}$$

$$\text{Άρα } K = \frac{1}{2} \cdot m \cdot v_o^2 = \frac{1}{2} \cdot 1 \cdot \sqrt{3}^2 = \frac{3}{2} = 1,5 \text{ J}$$

**Γ3.**

$$\Delta \overrightarrow{P}_2 = \overrightarrow{P}'_2 - \overrightarrow{P}_2 = \Delta P_2 = m_2 v_k - m_2 v_o \Rightarrow$$

$$\Delta P_2 = \frac{\sqrt{3}}{2} - \sqrt{3} \Rightarrow \Delta P_2 = -\frac{\sqrt{3}}{2} \text{ kg m/s} \Rightarrow$$

$$\Rightarrow |\Delta P_2| = \frac{\sqrt{3}}{2} \text{ kg m/s}$$

Με κατεύθυνση προς τα κάτω, προς τα αρνητικά.

**Γ4.** Για  $t = 0, x = 0,05 \text{ m}, A = 0,1 \text{ m}, v > 0$

$$x = A \eta \mu(\omega t + \varphi_0) \stackrel{t=0}{\Rightarrow} 0,05 = 0,1 \eta \mu \varphi_0 \Rightarrow \eta \mu \varphi_0 = \frac{1}{2} = \eta \mu \frac{\pi}{6}$$

$$\text{άρα } \left. \begin{array}{l} \varphi_0 = 2k\pi + \pi/6 \\ \varphi_0 = 2k\pi + 5\pi/6 \end{array} \right\} \begin{array}{l} 0 \leq \varphi_0 < 2\pi \\ \kappa=0 \end{array} \Rightarrow$$

$$\left. \begin{array}{l} \varphi_0 = \frac{\pi}{6} \text{ με } v > 0 \text{ δεκτή} \\ \varphi_0 = \frac{5\pi}{6} \quad v < 0 \end{array} \right\} \text{άρα } \varphi_0 = \frac{\pi}{6}$$

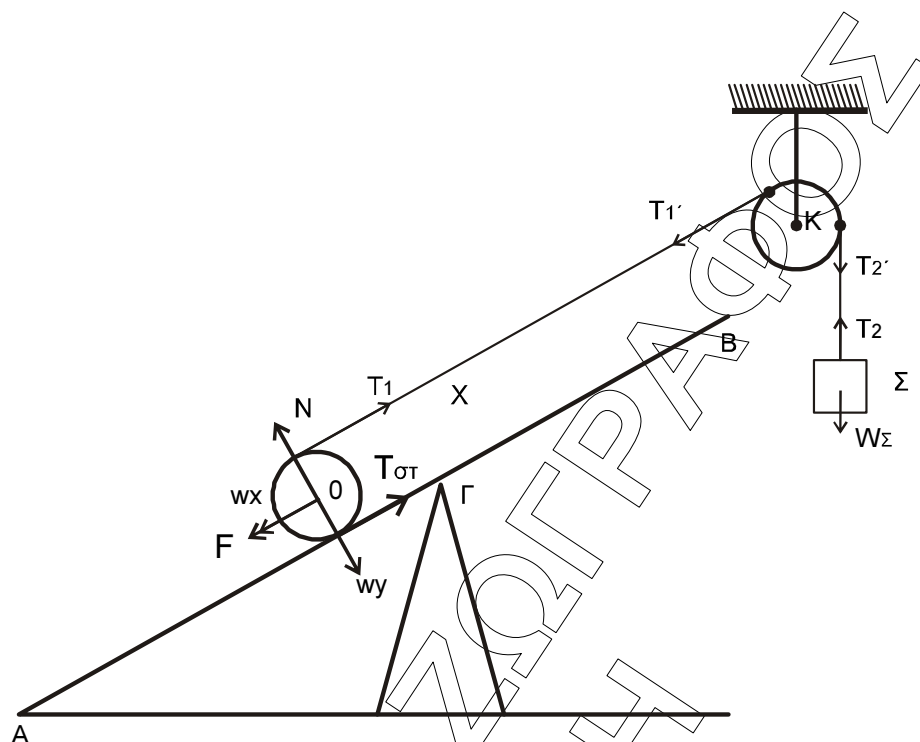
$$w = \sqrt{\frac{k}{m_1 + m_2}} = \sqrt{100} = 10 \text{ rad/s}$$

Άρα η απομάκρυνση είναι

$$x = 0,1 \eta \mu \left( 10t + \frac{\pi}{6} \right) \text{ (SI)}$$

## ΘΕΜΑ Δ

Δ1.



Σώμα (Σ):

$$\left. \begin{aligned} \Sigma F_y = 0 &\Rightarrow T_2 = W_\Sigma \\ T_2' = T_2 \end{aligned} \right\} T_2' = W_\Sigma \Rightarrow T_2' = 20 \text{ N}$$

Στην τροχαλία:

$$\Sigma \tau_{(K)} = 0 \Rightarrow T_1' R_T = T_2' R_T \Rightarrow T_1' = T_2' \Rightarrow T_1' = 20 \text{ N}$$

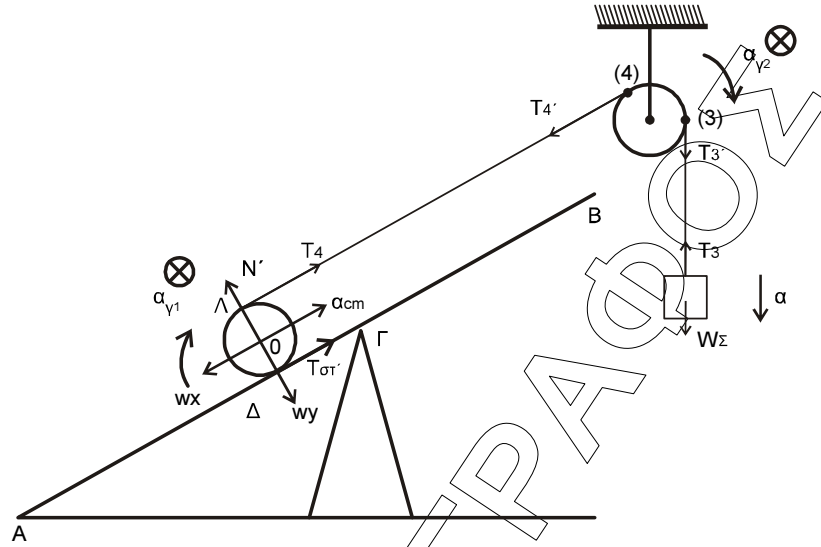
$$T_1 = T_1' = 20 \text{ N}$$

Στον κύλινδρο:

$$\Sigma \tau_{(O)} = 0 \Rightarrow T_1 R_K = T_{\sigma\tau} R_K \Rightarrow T_{\sigma\tau} = 20 \text{ N}$$

$$\Sigma F_x = 0 \Rightarrow F + W_x = T_1 + T_{\sigma\tau} \Rightarrow F = 40 - 10 \Rightarrow \underline{F = 30 \text{ N}}$$

Δ2.



$T_3 = T'_3$ ,  $T_4 = T'_4$  αβαρή νήματα

Για τον κύλινδρο

μεταφ:  $\Sigma F_x = m_k \cdot a_{cm} \Rightarrow T_4 + T'_{\sigma\tau} - W_x = m_k \cdot a_{cm}$  (1)

στροφ:  $\Sigma \tau_{(O)} = I_0 \cdot \alpha_{\gamma_1} \Rightarrow T_4 \cdot R_k - T'_{\sigma\tau} \cdot R_k = m_k \frac{R_k^2}{2} \alpha_{\gamma_1}$  (2)

Στην τροχαλία:

στροφ:  $\Sigma \tau_k = I_k \cdot \alpha_{\gamma_2} \Rightarrow T_3 \cdot R_T - T'_4 \cdot R_T = m_T \frac{R_T^2}{2} \alpha_{\gamma_2}$  (3)

Στο σώμα:

μεταφ:  $\Sigma F_y = m_\Sigma \cdot a \Rightarrow W_\Sigma - T_3 = m_\Sigma \cdot a$  (4)

η ταχύτητα του σώματος  $v_\Sigma = v_3$  άρα  $a_\Sigma = a \Rightarrow a = \alpha_{\gamma_2} R_T$  (5)

η ταχύτητα  $v_\Lambda = v_4$  άρα  $\alpha_\Lambda = \alpha_4 = \alpha_{\gamma_1} R_T$

όμως  $v_\Lambda = 2v_{cm}$  άρα  $\alpha_\Lambda = 2a_{cm}$

$v_\Delta = 0$  άρα  $v_{cm} = a \cdot R_k$  άρα  $\alpha_{cm} = a_{\gamma_1} R_k$  (7)

$2a_{cm} = a$  (6)

Λύνοντας:

Από (2), (7)  $\Rightarrow (T'_4 - T'_{\sigma\tau}) R_k = \frac{m_k R_k^2}{2} \frac{\alpha_{cm}}{R_k} \Rightarrow T'_4 - T_{\sigma\tau} = \frac{m_k}{2} \alpha_{cm} \stackrel{(m_k=2)}{\Rightarrow} T'_4 - T_{\sigma\tau} = \alpha_{cm}$  (8)

Από (3), (5)  $\Rightarrow (T_3 - T_4) R_T = \frac{m_T R_T^2}{2} \frac{\alpha}{R_T} \Rightarrow T_3 - T_4 = \frac{m_T}{2} \alpha \stackrel{(m_T=2)}{\Rightarrow} T_3 - T_4 = \alpha$  (9)

Από (8), (6)  $\Rightarrow T_4 - T_{\sigma\tau} = \frac{\alpha}{2}$  (10)

(4)  $20 - T_3 = 2\alpha$   
 (9)  $T_3 - T_4 = \alpha$  }  $20 - T_4 = 3\alpha$  (11)

$$\text{Από (1)(6)} \Rightarrow T_4 + T_{\sigma\tau} - m_K g \eta \mu \varphi = m_K \cdot a_{cm}$$

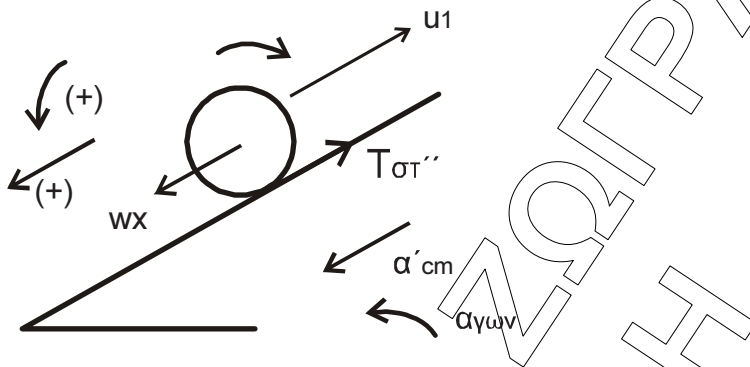
$$T_4 + T_{\sigma\tau} - 2 \cdot 10 \cdot \frac{1}{2} = 2 \frac{a}{2} \Rightarrow T_4 - T_{\sigma\tau} = 10 + a \quad (12)$$

$$\text{Από (10), (12)} \Rightarrow 2T_4 = 10 + \frac{3a}{2} \Rightarrow$$

$$2(20 - 3a) = 10 + \frac{3a}{2} \Rightarrow 40 - 6a = 10 + \frac{3a}{2} \Rightarrow a = 4 \frac{\text{m}}{\text{s}^2}$$

$$(6) \quad a_{cm} = \frac{a}{2} = 2 \frac{\text{m}}{\text{s}^2}.$$

Δ3.



$$v_1 = \alpha_{cm} \cdot t = 2 \cdot 0,5 = 1 \text{ m/s}$$

$$\Sigma F_x = M \cdot \alpha_{cm}$$

$$W_x - T_{\sigma\tau}'' = M_K \cdot \alpha_{cm}'$$

$$M_K \cdot g \cdot \eta \mu \varphi - T_{\sigma\tau}'' = M_K \cdot \alpha_{cm}' \quad (1)$$

$$\Sigma \tau = I_K \cdot \alpha'_{\gamma\omega\nu} \Rightarrow T_{\sigma\tau}'' \cdot R_K = \frac{M_K R_K^2}{2} \cdot \alpha'_{\gamma\omega\nu} \quad (2)$$

$$\text{κύλιση } \alpha'_{cm} = \alpha'_{\gamma\omega\nu} \cdot R_K \quad (3)$$

$$\text{Από (2) και (3)} \quad T_{\sigma\tau}'' \cdot R_K = M_K \frac{R_K^2}{2} \cdot \frac{\alpha'_{cm}}{R_K} \Rightarrow T_{\sigma\tau}'' = \frac{M_K \cdot \alpha'_{cm}}{2} = \frac{2 \cdot \alpha'_{cm}}{2} \Rightarrow$$

$$\Rightarrow T_{\sigma\tau}'' = \alpha'_{cm} \quad (4)$$

$$(1), (4) \quad 2 \cdot 10 \cdot \frac{1}{2} - \alpha'_{cm} = 2 \alpha'_{cm} \Rightarrow 10 = 3 \alpha'_{cm} \Rightarrow$$

$$\Rightarrow \alpha'_{cm} = \frac{10}{3} \text{ m/s}^2$$

$$v = v_1 - \alpha'_{cm} \cdot \Delta t \Rightarrow 0 = 1 - \frac{10}{3} \cdot \Delta t \Rightarrow \Delta t = \frac{3}{10} = 0,3 \text{ s}$$

$$t_{\text{STOP}} = 0,5 + \Delta t = 0,8 \text{ s}$$



Δ4.

$$S_{O\Lambda} = S_1 + S_2$$

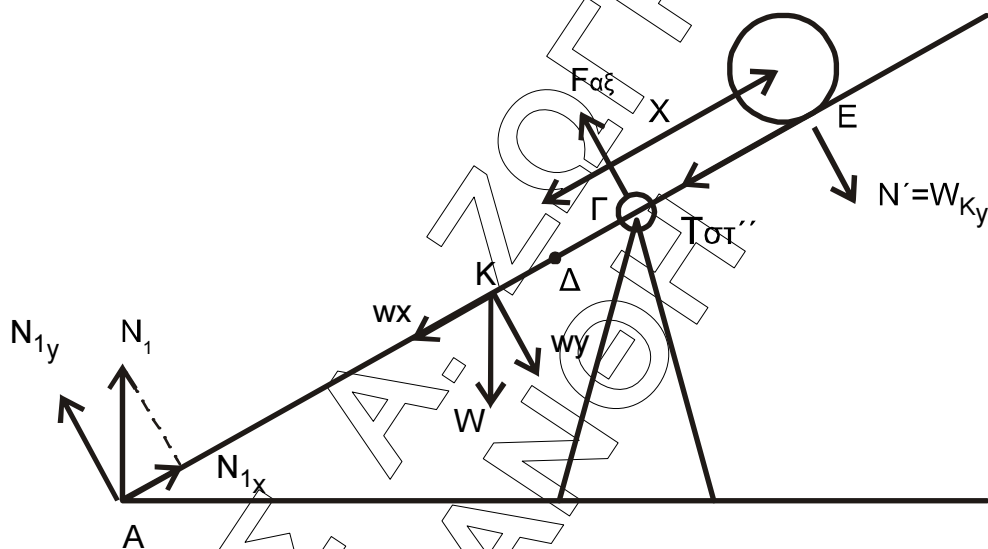
$$S_1 = \frac{1}{2} \alpha_{cm} t_1^2 = \frac{1}{2} \cdot 2 \cdot 0,5^2 = 0,25 \text{ m}$$

$$S_2 = v_1 \cdot \Delta t - \frac{1}{2} \cdot \alpha'_{cm} \cdot \Delta t^2 = 1 \cdot 0,3 - \frac{1}{2} \cdot \frac{10}{3} \cdot 0,3^2 =$$

$$= 0,3 - \frac{1}{2} \cdot \frac{10}{3} \cdot 0,3 \cdot 0,3 = 0,3 - \frac{1}{2} \cdot 0,3 = 0,15 \text{ m}$$

$$S_{O\Lambda} = 0,25 + 0,15 = 0,4 \text{ m}$$

Δ5. Οι δυνάμεις που ασκούνται στην σανίδα



$$\Sigma \tau_{(\Gamma)} = 0 \Rightarrow W_y(K\Gamma) - N_{1y}(A\Gamma) - W_{K_y'}(\Gamma E) = 0 \Rightarrow$$

$$\Rightarrow Mg \cdot \sigma\upsilon\nu\varphi(K\Gamma) - M_K \cdot g \cdot \sigma\upsilon\nu\varphi(\Gamma E) = N_1 \cdot \sigma\upsilon\nu\varphi(A\Gamma) \Rightarrow$$

$$\Rightarrow 20 \cdot 0,5 - 20 \cdot 0,2 = N_1 \cdot 2,5 \Rightarrow N_1 = \frac{6}{2,5} = 2,4 \text{ N}$$

Άρα  $N_1 > 0$  δέχεται δύναμη από το δάπεδο άρα δεν ανατρέπεται

Εναλλακτικά

$$\Sigma \tau_{(\Gamma)} = 0 \Rightarrow W_y(K\Gamma) - N_{1y}(A\Gamma) - W_{K_y'}(x - \Gamma\Delta) = 0 \Rightarrow$$

$$\Rightarrow Mg \cdot \sigma\upsilon\nu\varphi(K\Gamma) - M_K \cdot g \cdot \sigma\upsilon\nu\varphi(x - \Gamma\Delta) = N_1 \cdot \sigma\upsilon\nu\varphi(A\Gamma) \Rightarrow$$

$$\Rightarrow 20 \cdot 0,5 - 20 \cdot (x - 0,2) = N_1 \cdot 2,5 \Rightarrow 10 - 20x + 4 = N_1 \cdot 2,5 \Rightarrow$$

$$\Rightarrow 14 - 20x = 2,5N_1 \Rightarrow N_1 = 5,6 - 8x$$

για να μην ανατρέπεται πρέπει  $N_1 \geq 0 \Rightarrow 5,6 - 8x \geq 0 \Rightarrow x \leq 0,7 \text{ m}$

όμως  $0 \leq x \leq 0,4 \text{ m}$

άρα δεν ανατρέπεται.